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Master Thesis / Bachelor Thesis / Forschungspraxis

Greedy and Thresholding Based Methods versus Basis Pursuit for Sparse Recovery in Spherical Antenna Near-Field Measurements

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Measuring antennas in the near-field (NF) and performing a near-field to far-field transformation (NFFFT) can save costs and space over a measurement in the far-field (FF) [1]. The concept of NFFFTs is to represent the antenna under test by an expansion in equivalent sources (here spherical vector wave functions shall be used). The unknown expansion coefficients of the sources are then determined such that the measurements by a probe antenna are reproduced. This corresponds to solving a linear system of equations (LSE) [1]. However, in order to get a full rank LSE a large number of measurement samples has to be acquired, resulting in a long measurement time.

A theory allowing to reduce the number of samples without loss of accuracy is compressed sensing (CS). It defines a framework to uniquely solve an underdetermined LSE based on a priori knowledge about the matrix of the LSE and the sparsity level of the expansion coefficients [2]. One approach to actually find the solution is called basis pursuit (BP). Its strategy is to solve the optimization problem $\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{z}\in\mathbb{C}^N} \|\boldsymbol{z}\|_1$ s.t. $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{z}$, which corresponds to finding the approximately sparsest vector reproducing the measurements. While this approach usually requires the fewest number of samples, it has a high computational complexity compared to two other classes of reconstruction schemes. The first class contains so called greedy methods, such as orthogonal matching pursuit (OMP) or compressive sampling matching pursuit (CoSaMP). The other class comprises threshold based approaches like iterative hard thresholding (IHT) or hard thresholding pursuit (HTP) [3, 4, pp. 65ff].

In this work BP shall be compared to the greedy and thresholding based methods in the context of spherical NFFFT. Besides a comparison in terms of computational complexity, the minimum number of measurements for a successful recovery shall be determined by intensive numerical simulations. The stability and robustness against influences like measurement noise should be analyzed as well. To this end several so called phase transition diagrams need to be set up in a fashion similar to [5]. The results for BP are already available. The algorithms of the greedy and thresholding based methods have to be implemented in MATLAB, Julia or a similar program.

References

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