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Master Thesis / Bachelor Thesis / Forschungspraxis

Implementation and Performance Analysis of a Fast Basis Pursuit Solver in C/C++

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Compressed sensing (CS) defines a framework to uniquely solve an underdetermined linear system of equations (LSE) $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$. This is only possible having a priori information about the so called sensing matrix $\boldsymbol{A}$ and the vector of unknowns $\boldsymbol{x}$. Based on such information CS can provide theorems of when the LSE has a unique solution and how it is obtained [1].

One approach to actually find the solution is called basis pursuit (BP). Its strategy is to solve the optimization problem

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{z}\in\mathbb{C}^N} \|\boldsymbol{z}\|_1 \quad \text{s.t.} \quad \boldsymbol{y} = \boldsymbol{A}\boldsymbol{z}\,, \tag{1}$$

which corresponds to finding the sparsest vector fulfilling the LSE. For the case that the measurements are contaminated with noise, that is $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}$, it is often advantageous to generalize the minimization to

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{z}\in\mathbb{C}^N} \|\boldsymbol{z}\|_1 \quad \text{s.t.} \quad \left\|\boldsymbol{A}\boldsymbol{z} - \boldsymbol{y}\right\|_2 \leq \eta\,, \tag{2}$$

what is referred to as quadratically constrained basis pursuit. The choice of the parameter η depends on the noise level (e.g., for bounded noise $\|\boldsymbol{e}\|_2 \leq \eta$).

In this work the fast algorithm SPGL1 described in [2] to solve (1) and (2) shall be implemented in C/C++. Major advantage of SPGL1 is that it does not require the sensing matrix explicitly, but only products of the form $\boldsymbol{A}\boldsymbol{x}$ (i.e., the matrix is required only as an operator). This is crucial for the intended application of the solver in electromagnetic near-field to far-field transformations.

Besides a comparison with an available Matlab implementation [3] of the SPGL1 solver for verification purposes, a performance comparison in terms of accuracy, convergence and solution time with the commercial solvers Mosek [4] and Gurobi [5] shall be conducted.

References

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